Equilibrium in Combinatorial Public Projects for Agents with Complements

Extended Abstract

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ABSTRACT

The Combinatorial Public Projects Problem (CPPP) models a project allocation problem, in which agents have preferences over alternatives and a subset of projects is chosen to serve them collectively, in a way that maximizes the social welfare. We explore the problem from a mechanism design perspective, and study the inefficiency that arises due to a strategic behaviour. Previous work has considered this problem for bidders with complement-free valuations. We extend previous work by studying the Price of Anarchy in simple auctions for bidders with (restricted) complements. We also provide lower bounds for the Price of Anarchy in the classic Item Bidding auction for bidders with complement-free valuations.

There are plenty of situations in which a set of projects is chosen by an authority to collectively serve a group of people, e.g., when a city council chooses which roads to build for the benefit of the residents. The Combinatorial Public Projects Problem (CPPP) [2] models the above problem, which may be generalised as follows: a mechanism designer chooses a set S of at most k projects from a given set M of m public projects in favour of n strategic agents. Each agent has a private valuation function v that assigns a non-negative real number $v(S)$ to every bundle of projects $S \subseteq M$. The mechanism designer aims to maximize the social welfare $\text{SW}(S) = \sum_i v_i(S)$.

The CPPP has been studied under various valuation classes, e.g., [1, 3, 6]. The discussion is usually restricted to complement free valuations, meaning that the value of a set of items never exceeds the sum of values of its parts. Even for complement free valuations, every computationally efficient mechanism which is also truthful has a poor economic efficiency [2, 3]. These hardness results lead to the development of simple and efficient mechanisms which are not truthful. The inefficiency that arises due to strategic behaviour of the agents is commonly measured by the Price of Anarchy [1] (PoA).

One classic and simple mechanism is the Item Bidding with first prices mechanism [1], in which each agent simultaneously reports her valuation separately for each project. The mechanism chooses k projects, whose sum of valuations is maximal, and charges each agent her reported valuation for every selected project. This mechanism was studied in a complement free setting.

Limiting the discussion to a complement free setting misses common scenarios. For example, if the city council builds disconnected roads they will be useless. Hence, we would like to allow some degree of complementarity among projects. The main goal of the current work is to design a mechanism for agents with *restricted complementarities*, where a bundle can have a greater value than the sum of the values of its items. Item Bidding is not a suitable mechanism in this setting because single items might be non valuable without an entire bundle. To address this problem, we present a new mechanism, the Single Set Bidding mechanism. As far as we know this is the first work that designs and analyzes a mechanism for CPPP when agents have restricted complementarities. In the Single Set Bidding mechanism, each agent submits a bid t and a desired set of projects S. The mechanism designer iterates the bids in a descending order and adds each corresponding set to the allocated projects, as long as the union doesn't exceed k projects. Each agent pays her bid times $|S|$ if S was chosen, and 0 otherwise. We investigate the PoA of the Single Set Bidding mechanism as a function of a parameter associated with the underlying valuations, which expresses the degree of complementarity.

Several classes of valuations with restricted complements have been proposed in the literature [4, 5, 7, 8]. In the current work, we consider two hierarchies: Maximum over Constraint Homogeneous-d (MCH-d) [7] and Maximum over Single Minded-d (MSM-d). In both classes, the size of valuable bundles is bounded by a parameter d, which is associated with the inefficiency rate. We present an upper bound on the PoA of the Single Set Bidding mechanism w.r.t. Coarse Correlated Equilibrium (CCE)[2]. For the valuations class MSM-d, if $d \leq (1-c)k$ for some positive constant $c < 1$ than $PoA \leq \frac{dn}{c(1-e^{-1})k}$, otherwise $PoA \leq \frac{d^2 n}{(1-e^{-1})k}$. We present corresponding asymptotically tight lower bounds. For the valuations class MCH-d we show $PoA \leq \frac{dn}{1-e^{-1}}$. In the second part of this work we strengthen the results of [1] by presenting lower bounds for the PoA of Item Bidding first price mechanism for a wider set of cases than in [1]: $\frac{n}{k} - 1$ and n lower bounds on the PoA w.r.t. pure Nash equilibrium for unit demand [3] and for additive bidders, respectively.

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[1]The ratio between the optimal solution and the social welfare in the worst equilibrium.

[2]A more permissive equilibrium concept than the classic Pure Nash Equilibrium.

[3]A value of a bundle equals to the value of the most valuable item it contains.