A Binomial Distribution Model for TSP Based on Frequency quadrilaterals*

Yong Wang
School of Renewable Energy
P.O. Box 0108
China
wangyyong100@163.com

Jeffrey B. Remmel
Department of Mathematics
P.O. Box 0112
USA
jremmel@ucsd.edu

ABSTRACT

We study the symmetric travelling salesman problem (TSP) on the complete graph K_n via frequency graphs computed with frequency quadrilaterals. We introduce a binomial distribution model based on frequency quadrilaterals for TSP. When N frequency quadrilaterals containing an edge e are chosen at random to compute its frequency $F(e)$, the minimum frequency $F_{\min}$ of an edge e in the optimal Hamiltonian cycle edge (OHC) is bigger than $[4/3+4/(3(n\text{-}2))]N$. The experimental results for the real-world TSP instances illustrates the $F_{\min}\geq[3+2/(n\text{-}2)]N$. Moreover, we proves $F_{\min}$ tends to the maximum value $5N$ as n is big enough. This suggests a heuristic to reduce the number of edges that need to be considered in the search for the OHC by eliminating edges whose frequency is less than $F_{\min}$. We have run a variety of experiments that show that this is cases for TSP instances in TSPLIB [6].

• **Mathematics of computing → Discrete mathematics**; *Combinatorics*; *Probability and statistics*

KEYWORDS

Traveling salesman problem, Binomial distribution, Frequency quadrilateral, Sparse graph

1 INTRODUCTION

The traveling salesman problem (TSP) and its variations have been extensively studied by researchers in combinatorial optimization and computation complexity [1]. Here we will study the symmetric TSP on the complete graph K_n. We know that there are no polynomial-time algorithms for TSP unless P=NP [2]. Known exact algorithms for TSP have exponential-time bounds [3]. In the past decade, many researchers have studied the TSP on sparse graphs rather than on complete graph and have shown that more efficient algorithms exist to solve the TSP. For example, Xiao and Nagamochi [4] designed the $O(1.2312^n)$ time algorithm for TSP on 3-regular graphs. The goal of this research is to develop heuristics to start with a TSP on K_n and eliminate edges so that we are reduced to solving a TSP problem on a sparse graph by computing a frequency graph for the original TSP problem on K_n and eliminating edges with low frequency.

2 THE BINOMIAL DISTRIBUTION MODEL

Given 4 vertices ABCD in K_n, we consider the number of times that an edge e appears in an optimal path of length 4 included in the complete graph on ABCD. We call this the frequency of e.

We show that there are six possible frequency quadrilaterals for ABCD in K_n which are determined by the order of the three distance sums: d(A,B)+d(C,D), d(A,C)+d(B,D), d(A,D)+d(B,C) in the ABCD [5]. In the six possible frequency quadrilaterals for ABCD, a given e of ABCD appears with frequency 1, 3 and 5 twice. Thus, we study a model where the probability that an edge e has frequency 1, 3 and 5 in a frequency quadrilateral is $p_1(e)=p_3(e)=p_5(e)=1/3$. We combine $p_3(e)$ and $p_5(e)$ together as $p_{3,5}(e)=2/3$. For the edges in the OHC, we can show that the probability $p_{3,5}(e)=2/3+2/(3(n\text{-}2))$. If we choose N frequency quadrilaterals containing e in K_n, the probability that there are k frequency quadrilaterals where e has frequency 3 or 5 conforms to the binomial distribution model.

$$P(X=k)=\binom{N}{k}\left(\frac{2}{3}\right)^{k}\left(\frac{1}{3}\right)^{N-k} \quad (1)$$

According to the binomial distribution model, we derive the minimum frequency $F_{\min}$ of the edge in the OHC is $(4/3+4/(3(n\text{-}2)))N$. In average case, $F_{\min}\geq(3+2/(n\text{-}2))N$. This suggests that we can eliminate the edges with frequency below $F_{\min}$ and generate a graph with a small number of edges for TSP which still contains the original OHC. We have done experiments for the TSP instances in TSPLIB [6]. The results show that in many cases, a sparse graph with $O(n \log(n))$ edges can be computed which still contains the original OHC. In other experiments, we iterated the process of eliminating edges with low frequencies to produce even sparser graphs which still contain the original OHC. In such cases, we have reduced the original TSP to the problem of finding the OHC in a sparse graph where a variety of improved algorithms are available to resolve such TSP.

REFERENCES

[1] Gutin G. and Punnen A. P. 2007. *The traveling salesman problem and its variations*, New York, Springer US, Vol.12.
[2] Karp, R.M. 1975. On the computational complexity of combinatorial problems. *Networks* 5, 1, 45–68.
[3] Björklund A. 2010. Determinant sums for undirected Hamiltonicity. *Proceedings of the 2010 IEEE 51st Annual Symposium on Foundations of Computer Science*, Las Vegas, USA, 173–182.
[4] Xiao M. and Nagamochi H. 2016. An exact algorithm for TSP in degree-3 graphs via circuit procedure and amortization on connectivity structure. *Algorithmica* 74, 2, 713–741.
[5] Wang Y. and Remmel J.B. 2016. A binomial distribution model for the traveling salesman problem based on frequency quadrilaterals. *Journal of Graph Algorithms and Applications* 20, 2, 411–434.
[6] Reinelt G. http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/.

ACKNOWLEDGMENTST

This work was partially supported by the Fundamental Research Funds for the Central Universities (No.2015ZD10) and by the Project of Discrimination of OHC edges for TSP, Beijing.